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On Fuzzy Semi-P-spaces and Related Concepts

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Abstract: In this paper the notion of fuzzy semi-P-spaces is introduced and studied. It is established that the class of fuzzy semi-P-spaces lies between the classes of fuzzy-P-spaces and fuzzy almost-P-spaces. It is established that fuzzy fuzzy σ-nowhere dense sets in fuzzy Semi-P-spaces are fuzzy Semi-closed sets and fuzzy G_δ-sets and fuzzy F_σ-sets are fuzzy σ-nowhere sets in fuzzy hyperconnected and semi-P-spaces are fuzzy dense sets. Also it is found that fuzzy residual sets in fuzzy globally disconnected and fuzzy Semi-P-spaces are fuzzy semi-open sets and fuzzy G_δ-sets in fuzzy perfectly disconnected and fuzzy Semi-P-spaces are fuzzy pre-open sets. Also it is established that fuzzy hyperconnected and semi-P-spaces are fuzzy irresolvable spaces. The conditions for fuzzy semi- P-spaces to become fuzzy σ-Baire spaces and fuzzy Baire spaces are obtained. The conditions for the fuzzy semi-P-spaces to become fuzzy strongly irresolvable spaces are also obtained in this paper.

Keywords: Fuzzy G_δ-set, Fuzzy F_σ-set, Fuzzy Semi-open Set, Fuzzy Almost P-space, Fuzzy σ-Baire Space, Fuzzy Baire Space, Fuzzy Perfectly Disconnected Space

1. Introduction

The concept of fuzzy sets as a new approach for modeling uncertainities was introduced by L. A. Zadeh [21] in the year 1965. The concept of fuzzy topological spaces was introduced by C. L. Chang [4] in 1968. Chang's works paved the way for the subsequent tremendous growth of the numerous fuzzy topological concepts. The concept of fuzzy P-spaces in fuzzy setting was introduced by G. Balasubramanian [7]. Fuzzy Almost P-spaces and Fuzzy *GID* – spaces were introduced and studied by G. Thangaraj and C. Anbazhagan in [13, 14].

In this paper, the notion of fuzzy semi-P-space is introduced by means of fuzzy semi-openness of fuzzy G_δ-sets. Several characterizations of fuzzy semi-P-spaces are established. It is established that the class of fuzzy semi-P-spaces lies between the classes of fuzzy P-spaces and fuzzy almost P-spaces. Also it is obtained that fuzzy F_σ –sets in fuzzy semi-P-spaces are fuzzy cs dense sets but not fuzzy dense sets and fuzzy F_σ-sets in fuzzy hyperconnected and fuzzy semi-P-spaces, are fuzzy σ-nowhere dense sets. Also it is obtained that fuzzy residual sets in fuzzy globally disconnected and fuzzy semi-P-spaces, are fuzzy semi-open sets. The conditions for the fuzzy hyperconnected and fuzzy semi-P-spaces to become fuzzy σ-Baire spaces and for the fuzzy hyper connected, fuzzy open hereditarily irresolvable and fuzzy semi-P-spaces to become fuzzy Baire spaces are obtained. The conditions for the fuzzy semi-P-spaces to become fuzzy strongly irresolvable spaces and for the fuzzy semi-P-spaces to become fuzzy P-spaces are also obtained in this paper.

2. Preliminaries

Some basic notions and results used in the sequel are given in order to make the exposition self - contained. In this work by (X,T) or simply by X, we will denote a fuzzy topological space due to Chang (1968). Let X be a non - empty set and I the unit interval [0,1]. A fuzzy set λ in X is a mapping from X into I. The fuzzy set 0_X is defined as $0_X(x) = 0$, for all $x \in X$ and the fuzzy set 1_X is defined as $1_X(x) = 1$, for all $x \in X$.

Definition 2.1 [4]: Let (X, T) be a fuzzy topological space and λ be any fuzzy set in (X,T). The interior, the closure and the complement of λ are defined respectively as follows:

(i). $\mathrm{int}(\lambda) = \vee \{ \mu / \mu \leq \lambda, \mu \in \mathrm{T}\}$;

(ii). $\mathrm{cl}(\lambda) = \wedge \{ \mu / \lambda \leq \mu, 1-\mu \in \mathrm{T}\}$.

(iii). $\lambda'(x) = 1-\lambda(x)$, for all $x \in X$.

For a family $\{\lambda_i / i \in I\}$ of fuzzy sets in (X, T), the union $\psi = \vee_i (\lambda_i)$ and intersection $\delta = \wedge_i (\lambda_i)$, are defined respectively as

(iv). $\psi(x) = sup_i \{\lambda_i(x) / x \in \mathrm{X}\}$

(v). $\delta(x) = \inf_i \{\lambda_i(x) / x \in \mathrm{X}\}$.

Lemma 2.1 [1]: For a fuzzy set λ of a fuzzy topological space X,

(i).$1 - int(\lambda) = cl(1-\lambda)$ and (ii). $1 - cl(\lambda) = int(1-\lambda)$.

Definition 2.2: A fuzzy set λ in a fuzzy topological space (X,T) is called an

(i). fuzzy regular-open if $\lambda = \mathrm{int\ cl}(\lambda)$ and fuzzy regular-closed if $\lambda = \mathrm{cl\ int}(\lambda)$ [1].

(ii). fuzzy pre-open if $\lambda \leq \mathrm{int\ cl}(\lambda)$ and fuzzy pre-closed if $\mathrm{cl\ int}(\lambda) \leq \lambda$ [3].

(iii). fuzzy semi-open if $\lambda \leq \mathrm{cl\ int}(\lambda)$ and fuzzy semi-closed if $\mathrm{int\ cl}(\lambda) \leq \lambda$ [1].

(iv). fuzzy β-open if $\lambda \leq \mathrm{cl\ int\ cl}(\lambda)$ and fuzzy β–closed if $\mathrm{int\ cl\ int}(\lambda) \leq \lambda$ [3].

(v). fuzzy G_δ -set if $\lambda = \wedge_{i=1}^{\infty}(\lambda_i)$, where $\lambda_i \in T$ for $i \in I$ [2].

(vi). fuzzy F_σ -set if $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$, where $1-\lambda_i \in T$ for $i \in I$ [2].

Definition 2.3: The fuzzy set λ in an fuzzy topological space (X,T), is called an

(i). fuzzy dense set if there exists no fuzzy closed set μ in (X,T) such that $\lambda < \mu < 1$. That is $cl(\lambda) = 1$, in (X,T) [8].

(ii). fuzzy nowhere dense set if there exists no non-zero fuzzy open set μ in (X,T) such that $\mu < cl(\lambda)$. That is, $int\ cl(\lambda) = 0$, in (X,T) [8].

(iii). fuzzy first category set if $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$, where (λ_i)'s are fuzzy nowhere dense sets in (X,T). Any other fuzzy set in (X,T) is said to be of fuzzy second category [8].

(iv). fuzzy somewhere dense set if there exists a non-zero fuzzy open set μ in (X,T) such that $\mu < cl(\lambda)$. That is, $int\ cl(\lambda) \neq 0$, in (X,T) [9].

(v). fuzzy cs dense set if $1-\lambda$ is an fuzzy somewhere dense set in (X, T) [16].

(vi). fuzzy σ- nowhere dense set if λ is a fuzzy F_σ –set with $\mathrm{int}(\lambda) = 0$, in (X,T) [12].

(vii). fuzzy residual set if $1-\lambda$ is an fuzzy first category set in (X,T) [11].

(viii). fuzzy σ-boundary set if $\lambda = \vee_{i-1}^{\infty}(\mu_i)$, where $\mu_i = cl(\lambda_i) \wedge (1-\lambda_i)$ and (λ_i)'s are fuzzy regular open sets in (X, T) [15].

(ix). fuzzy co - σ boundary set if $1-\lambda$ is an fuzzy σ-boundary set in (X,T) [15].

(x). Fuzzy simply open set if $\mathrm{Bd}(\lambda)$ is a fuzzy nowhere dense set in (X,T). That is, λ is a fuzzy simply open set in (X,T) if $[cl(\lambda) \wedge cl(1-\lambda)]$ is a fuzzy nowhere dense set in (X,T) [18].

Definition 2.4: A fuzzy topological space (X,T) is called an

(i). fuzzy P-space if each fuzzy G_δ-set in (X,T) is fuzzy open in (X,T) [7].

(ii). fuzzy almost P - space if for every non- zero fuzzy G_δ-set λ in (X,T). $int(\lambda) \neq 0$ in (X,T) [13].

(iii). fuzzy globally disconnected space if each fuzzy semi-open set in (X,T) is fuzzy open in (X,T) [17].

(iv). fuzzy hyperconnected space if every non- null fuzzy open subset of (X, T) is fuzzy dense in (X, T) [6].

(v). fuzzy perfectly disconnected space if for any two non – zero fuzzy sets λ and μ defined on X with $\lambda \leq 1-\mu$, $cl(\lambda) \leq 1-cl(\mu)$, in (X,T) [18].

(vi). fuzzy strongly irresolvable space if $cl\ int(\lambda) = 1$, for each fuzzy dense set λ in (X, T) [19].

(vii). fuzzy submaximal space if for each fuzzy set λ in (X, T) such that $cl(\lambda) = 1$, then $\lambda \in T$ [2].

(viii). fuzzy Baire space if $int(\vee_{i=1}^{\infty}(\lambda_i)) = 0$, where (λ_i)'s are fuzzy nowhere dense sets in (X,T) [11].

(ix). fuzzy σ -Baire space $int(\vee_{i=1}^{\infty}(\lambda_i)) = 0$, where (λ_i)'s are fuzzy σ – nowhere dense sets in (X,T) [12].

(x). fuzzy open hereditarily irresolvable space if $int\ cl(\lambda) \neq 0$, for any non-zero fuzzy set λ defined on X, then $int(\lambda) \neq 0$, in (X,T) [10].

(xi). fuzzy resolvable space if there exists an fuzzy dense set λ in (X,T) such that $cl(1-\lambda) = 1$. Otherwise (X,T) is called an fuzzy irresolvable space [10].

(xii). Fuzzy *GID* space if for each fuzzy dense and fuzzy G_δ-set λ in (X,T), $cl\ int(\lambda) = 1$, in (X, T) [14].

Definition 2.5 [20]: Let (X, T) be any fuzzy topological space and λ be any fuzzy set in (X, T). The fuzzy semi-closure and the fuzzy semi-interior of λ are defined as follows:

(1) scl-$(\lambda) = \wedge\{\mu / \lambda \leq \mu$, μ is fuzzy semi closed in (X,T)$\}$;

(2) sint - $(\lambda) = \vee\{\mu / \mu \leq \lambda$, μ is fuzzy semi open in (X,T)$\}$.

Theorem 2.1 [17]: If (X,T) is the fuzzy globally disconnected space, then for any fuzzy semi-closed set λ in (X,T), $cl(1-\lambda) = 1 - cl\ int(\lambda)$, in (X,T).

Theorem 2.2 [13]: If γ is an fuzzy residual set in an fuzzy almost P-space (X,T), then there exists an fuzzy G_δ-set μ in (X,T) such that $\mu \leq \gamma$.

Theorem 2.3 [16]: If λ is an fuzzy somewhere dense set in an fuzzy topological space (X, T), then there exist a fuzzy regular closed set η in (X, T) such that $\eta \leq cl(\lambda)$.

Theorem 2.4 [16]: If λ is an fuzzy somewhere dense set in a fuzzy hyperconnected space (X, T), then λ is an fuzzy dense set in (X, T).

Theorem 2.5 [17]: If λ is an fuzzy residual set in the globally disconnected space (X,T), then λ is an fuzzy G_δ-set in (X,T).

Theorem 2.6 [11]: If (X, T) is an fuzzy Baire space, then (X, T) is an fuzzy second category space.

Theorem 2.7 [20]: If λ is an fuzzy σ-boundary set in the fuzzy topological space (X,T), then λ is an fuzzy F_σ–set in (X,T).

Theorem 2.8 [18]: If λ is an fuzzy σ-nowhere dense set in the fuzzy perfectly disconnected space (X,T), then there exists an fuzzy G_δ - set η in (X,T) such that $\lambda \leq int\,(1-\eta)$.

Theorem 2.9 [18]: If λ is an fuzzy co -σ -boundary set in the fuzzy perfectly disconnected space (X,T), then there exists an fuzzy G_δ-set δ in (X,T) such that $\lambda \leq \delta$.

Theorem 2.10 [18]: If λ is a fuzzy semi-closed set in a fuzzy perfectly disconnected space(X,T), then λ is a fuzzy pre-closed set in (X,T).

Theorem 2.11 [14]: Let (X,T) be a fuzzy topological space. Then the following are equivalent:

(i). (X, T) is a fuzzy *GID* space.

(ii). Each fuzzy dense and fuzzy G_δ-set in (X,T), is fuzzy semi-open in (X, T).

Theorem 2.12 [17]: If λ is a fuzzy semi-open and fuzzy dense set in the fuzzy globally disconnected space (X,T), then λ is the fuzzy simply open set in (X,T).

3. Fuzzy Semi-P-spaces

Definition 3.1: A fuzzy topological space (X,T) is called an fuzzy semi- P-space if each fuzzy G_δ-set in (X, T) is an fuzzy semi-open set in (X,T). That is (X,T) is an fuzzy semi- P-space if whenever λ is an fuzzy G_δ-set in (X,T), then $\lambda \leq cl\ int(\lambda)$, in (X, T).

Example 3.1: Let X= {a, b, c}. Let I=[0,1]. The fuzzy sets α, β and γ are defined on X as follows:

α: X→I is defined by α(a)=0.6; α (b) =0.3; α (c)=0.4,

β: X→I is defined by β(a)=0.4; β (b) =0.7; β(c)=0.6,

γ: X→ I is defined by γ(a)=0.5; γ (b) =0.5; γ (c)=0.5,

Then $T = \{0, \alpha, \beta, \gamma, \alpha\vee\beta, \alpha\vee\gamma, \beta\vee\gamma, \alpha\wedge\beta, \alpha\wedge\gamma, \beta\wedge\gamma, 1\}$ is the fuzzy topology on X. On computation, $\gamma = (\alpha\vee\beta) \wedge (\alpha\vee\gamma) \wedge (\beta\vee\gamma)$, $\alpha\wedge\beta = \alpha\wedge\beta\wedge\gamma \wedge (\alpha\wedge\beta) \wedge (\alpha\wedge\gamma) \wedge (\beta\wedge\gamma)$. And thus γ and $\alpha\wedge$ β are fuzzy G_δ-sets in (X, T). Also cl int (γ) = cl (γ) $= 1-\gamma$ and $cl\ int(\alpha\wedge\beta) = cl(\alpha\wedge\beta) = 1-(\alpha\vee\beta)$. Now $\gamma \leq 1-\gamma$ and $\alpha\wedge\beta \leq 1-(\alpha\vee\beta)$, implies that $\gamma \leq cl\ int(\gamma)$ and (α ∧ β) ≤ cl $int(\alpha\wedge\beta)$, in (X,T). Hence the fuzzy G_δ -sets γ and $\alpha\wedge\beta$ are fuzzy semi-open sets in (X, T), implies that (X,T) is an fuzzy semi- P -space.

Proposition 3.1: If λ is an fuzzy G_δ-set in the fuzzy semi-P-space (X,T), then

(i). $int(\lambda) \neq 0$, in (X,T).

(ii). λ is an fuzzy somewhere dense set in (X,T).

(iii). $cl\ int(\lambda) = cl(\lambda)$, in (X,T).

(iv). λ is an fuzzy β –open set in (X,T).

Proof:

(i). Let λ be an fuzzy G_δ-set in (X,T). Since (X,T) is an fuzzy semi-P-space, the fuzzy G_δ-set λ is an fuzzy semi-open set (X,T). Then, $\lambda \leq cl\ int\ (\lambda)$, in (X,T). This implies that $int(\lambda) \neq 0$, in (X,T). [otherwise, if $int(\lambda) = 0$, $cl\ int(\lambda) = cl\ (0) = 0$ and this results in $\lambda = 0$, a contradiction].

(ii). By (i), $int(\lambda) \neq 0$, in (X,T). Since $int(\lambda) \leq int\ cl(\lambda)$, $int\ cl(\lambda) \neq 0$, in (X,T). Hence λ is an fuzzy somewhere dense set in (X,T).

(iii). Since (X,T) is an fuzzy semi-P-space, the fuzzy G_δ-set $\lambda$is an fuzzy semi-open set in (X,T). Then, $\lambda \leq cl\ int(\lambda)$, in (X,T). This implies that $cl(\lambda) \leq cl\ [cl\ int(\lambda)]$ and $cl(\lambda) \leq cl\ int\ (\lambda)$. But $cl\ int(\lambda) \leq cl(\lambda)$. Thus, $cl\ int(\lambda) = cl(\lambda)$, in (X,T).

(iv). Since (X,T) is an fuzzy semi- P-space, for the fuzzy G_δ-set λ in (X,T), $\lambda \leq cl\ int(\lambda)$. Now $cl\ int\ (\lambda) \leq cl\ int\ cl(\lambda)$, implies that $\lambda \leq cl\ int\ cl(\lambda)$, in (X,T) and hence λ is an fuzzy β-open set in (X,T).

Proposition 3.2: If λ is an fuzzy dense and fuzzy G_δ-set in the fuzzy semi-P-space (X,T), then $cl\ int(\lambda) = 1$, in (X,T).

Proof: Let λ be an fuzzy dense and fuzzy G_δ -set in (X,T). Since (X,T) is an fuzzy semi-P-space, the fuzzy G_δ-set λ is an fuzzy semi-open set in (X,T). Then, $\lambda \leq cl\ int\ (\lambda)$, in (X,T). This implies that $cl(\lambda) \leq cl\ [cl\ int(\lambda)]$. Thus, $1 \leq cl\ int(\lambda)$. That is, $cl\ int\ (\lambda) = 1$, in (X,T).

Proposition 3.3: If (X,T) is an fuzzy semi-P-space, then (X,T) is an fuzzy almost P-space.

Proof: Let λ be an fuzzy G_δ-set in (X,T). Since (X,T) is the fuzzy semi-P-space, by the proposition 3.1(i), for the fuzzy G_δ-set λ in (X,T), $int(\lambda) \neq 0$. Hence, for the fuzzy G_δ-set λ in (X,T), $int(\lambda) \neq 0$, implies that (X,T) is an fuzzy almost P-space.

Proposition 3.4: If λ is an fuzzy residual set in the fuzzy semi-P-space (X,T), then:

(i). There exists an fuzzy semi-open set μ in (X,T) such that $\mu \leq \lambda$.

(ii). $s\text{-}int(\lambda) \neq 0$, in (X,T).

Proof:

(i). Let λ be an fuzzy residual set in (X,T). Since (X,T) is an fuzzy semi-P-space, by the proposition 3.3, (X,T) is an fuzzy almost P-space. Then, by the theorem 2.2, for the fuzzy residual set λ in (X,T), there exists an fuzzy G_δ-set μ in (X,T) such that $\mu \leq \lambda$. Since (X,T) is an fuzzy semi-P-space, the fuzzy G_δ-set μ in (X,T), is an fuzzy semi-open in (X,T) Thus, there exists an fuzzy semi-open set μ in (X,T) such that $\mu \leq \lambda$.

(ii). From (i), for the fuzzy residual set λ in (X,T), there exists an fuzzy semi-open setμin (X,T) such that$\mu \leq \lambda$ and hence $s\text{-}int\ (\lambda) \neq 0$, in (X,T).

Proposition 3.5: If λ is an fuzzy σ-nowhere dense set in the fuzzy semi-P-space (X,T), then λ is the fuzzy semi-closed set in (X,T).

Proof: Let λ be an fuzzyσ-nowhere dense set in (X,T). Then, λ is the fuzzy F_σ-set in (X,T) such that $int(\lambda) = 0$. Then, $1-\lambda$ is the fuzzy G_δ-set in (X,T). Since (X,T) is the fuzzy semi-P-space, the fuzzy G_δ-set $(1-\lambda)$ is the fuzzy semi-open set in (X,T) and thus λ is the fuzzy semi-closed set in (X,T).

Proposition 3.6: If λ is an fuzzy σ-nowhere dense set in the fuzzy semi-P-space (X,T), then

(i). There exists an fuzzy semi-closed η in (X,T) such that $\lambda \leq \eta$.

(ii). $s\text{-}cl\ (\lambda) \neq 1$, in (X,T).

Proof:

(i). Let λ be an fuzzy σ-nowhere dense set in (X,T). Then, λ is the fuzzy F_σ- set with $int(\lambda) = 0$,in (X,T) and thus $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$,where $1-\lambda_i \in T$, for$i \in I$. Now $int(\lambda) = 0$, implies that $int(\vee_{i=1}^{\infty}(\lambda_i)) = 0$, in (X,T). But

$\vee_{i=1}^{\infty} int\ (\lambda_i) \leq int(\ \vee_{i=1}^{\infty}(\lambda_i))$ and thus $\vee_{i=1}^{\infty} int\ (\lambda_i) = 0$. This implies that $int\ (\lambda_i) = 0$. Since (λ_i)'s are fuzzy closed sets in (X,T), $int\ cl(\lambda_i) = int\ (\lambda_i) = 0$ and then (λ_i)'s are fuzzy nowhere dense sets in (X,T). Thus, $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$, where (λ_i)'s are fuzzy nowhere dense sets in (X,T), implies that λ is the fuzzy first category set in (X,T) and thus $1-\lambda$ is an fuzzy residual set in (X,T). Since (X,T) is the fuzzy semi-P-space, by the proposition 3.4 (i), there exists an fuzzy semi-open set μ in (X,T) such that $\mu \leq 1-\lambda$. Then, $\lambda \leq 1-\mu$, in (X,T). Since μ is an fuzzy semi-open set in (X,T), $1-\mu$ is an fuzzy semi-closed set in (X,T). Let $\eta = 1-\mu$. Thus, there exists an fuzzy semi-closed set η in (X,T) such that $\lambda \leq \eta$.

(ii). From (i), for the fuzzy σ-nowhere dense set λ in (X,T), there exists an fuzzy semi-closed set η in (X,T) such that $\lambda \leq \eta$ and hence $s\text{-}cl\ (\lambda) \neq 1$, in (X,T).

Proposition 3.7: If λ is an fuzzy G_δ-set in the fuzzy semi-P-space (X,T), then there exist an fuzzy regular closed set η in (X, T) such that $\eta \leq cl\ (\lambda)$.

Proof: Let λ be an fuzzy G_δ-set in (X,T). Since (X,T) is an fuzzy semi-P-space, by the proposition 3.1 (ii), the fuzzy G_δ-set λ is an fuzzy somewhere dense set in (X,T). Then, by the theorem 2.3, there exist an fuzzy regular closed set η in (X, T) such that $\eta \leq cl\ (\lambda)$.

Remarks 3.1: In view of the propositions 3.1(iii) and 3.7, one will have the following result: "If λ is an fuzzy G_δ-set in the fuzzy semi-P-space (X,T), then there exists an fuzzy regular closed set η in (X, T) $\eta \leq cl\ int(\lambda)$, in (X,T)".

Proposition 3.8: If λ is an fuzzy F_σ –set in the fuzzy semi-P-space (X, T), then

(i). λ is an fuzzy cs dense set in (X, T).

(ii). $cl(\lambda) \neq 1$ in (X, T).

Proof:

(i). Let λ be an fuzzy F_σ –set in (X,T). Since (X,T) is the fuzzy semi-P-space, by the proposition 3.3, (X,T) is the fuzzy almost P-space. Now $1-\lambda$ is the fuzzy G_δ-set in (X, T). Since (X, T) is the fuzzy almost P-space, for the fuzzy G_δ -set $1-\lambda$, $int\ (1-\lambda) \neq 0$, in(X, T). Then, $int\ (1-\lambda) \leq int\ cl\ (1-\lambda)$, implies that $int\ cl\ (1-\lambda) \neq 0$, in (X, T) and thus $1-\lambda$ is the fuzzy somewhere dense set in (X, T). Hence λ is an fuzzy cs dense set in (X, T).

(ii). Let λ be an fuzzy F_σ-set in (X,T). Since (X,T) is the fuzzy semi-P-space, by the proposition 3.3, (X,T) is the fuzzy almost P-space. Since (X, T) is the fuzzy almost P-space, for the fuzzy G_δ-set $1-\lambda$, $int\ (1-\lambda) \neq 0$, in (X, T). By the lemma 2.1, $1\text{-}cl(\lambda) = int\ (1-\lambda) \neq 0$ in (X, T). Hence $cl(\lambda) \neq 1$, in (X, T).

Remarks 3.2: In view of the proposition 3.8 (i), one will have the following result: "Fuzzy F_σ-sets in fuzzy semi-P-spaces are fuzzy cs dense sets but not fuzzy dense sets".

Proposition 3.9: If λ is an fuzzy σ-boundary set in the fuzzy topological space (X,T), then

(i). λ is an fuzzy cs dense set in (X, T).

(ii). $cl(\lambda) \neq 1$ in (X, T).

Proof: The proof follows from the proposition 3.7 and the theorem 2.7.

4. Fuzzy Semi-P-spaces and Other Fuzzy Topological Spaces

Proposition 4.1: If λ is an fuzzy G_δ -set in the fuzzy hyperconnected and fuzzy semi-P-space (X,T), then

(i). λ is an fuzzy dense set in (X, T).

(ii). $int\ (1-\lambda) = 0$, in (X, T).

Proof:

(i). Let λ be an fuzzy G_δ-set in (X, T). Since (X,T) is an fuzzy semi-P-space, by the proposition 3.1(ii), the fuzzy G_δ-set λ is an fuzzy somewhere dense set in (X,T). Also since (X,T) is the fuzzy hyperconnected space, by the theorem 2.4, the fuzzy somewhere dense set λ is an fuzzy dense set in (X, T).

(ii). From (i), for the fuzzy G_δ-set λ in(X, T), $cl(\lambda) = 1$ and then by the lemma 2.1, $int\ (1-\lambda) = 1-cl(\lambda) = 1-1 = 0$, in (X, T).

The following proposition shows that fuzzy F_σ-sets in fuzzy hyperconnected and fuzzy semi-P-spaces, are fuzzy σ-nowhere dense sets.

Proposition 4.2: If λ is an fuzzy F_σ -set in the fuzzy hyperconnected and fuzzy semi-P-space (X,T), then λ is an fuzzy σ-nowhere dense set in (X, T).

Proof: Let λ be an fuzzy F_σ –set in the fuzzy hyperconnected and fuzzy semi-P-space (X, T). Now $1-\lambda$ is the fuzzy G_δ-set in (X, T). Then, by the proposition 4.1(ii), $int(1-[1-\lambda])= 0$, in (X, T) and then $int(\lambda) = 0$, in (X, T). Thus, λ is an fuzzy F_σ-set, in (X, T) such that $int(\lambda) = 0$. Hence λ is the fuzzy σ-nowhere dense set in (X, T).

Remark 4.1: In view of the above propositions one will have the following results:

"Fuzzy G_δ-sets are fuzzy dense sets and fuzzy F_σ –sets are fuzzy σ-nowhere dense sets in fuzzy hyperconnected and fuzzy semi-P-spaces".

The following proposition gives the condition for the fuzzy hyperconnected and fuzzy semi- P-spaces to become fuzzy σ-Baire spaces.

Proposition 4.3: If $int\ [\vee_{i=1}^{\infty}(\lambda_i)] = 0$, where $(\lambda_i)'$s are fuzzy F_σ- sets in the fuzzy hyperconnected and fuzzy semi-P-space (X, T), then (X, T) is an fuzzy σ–Baire space.

Proof: Let $(\lambda_i)'$s (I = 1 to ∞) be the fuzzy F_σ-sets in (X, T). Since (X,T) is the fuzzy hyperconnected and fuzzy semi-P-space, by the proposition 4.2, $(\lambda_i)'$s are fuzzy σ-nowhere dense sets in (X,T). Then, from the hypothesis, $int\ [\vee_{i=1}^{\infty}(\lambda_i)] = 0$, where $(\lambda_i)'$s are fuzzy σ-nowhere dense sets in (X, T). This implies that (X, T)is an fuzzy σ-Baire space.

Proposition 4.4: If λ is an fuzzy residual set in the fuzzy globally disconnected and fuzzy semi-P-space (X,T), then λ is an fuzzy semi-open set in (X,T).

Proof: Let λ be an fuzzy residual set in (X,T). Since (X,T) is an fuzzy globally disconnected space, by the theorem 2.5, the fuzzy residual set λ is an fuzzy G_δ-set in (X,T). Since

(X,T) is an fuzzy semi-P-space, the fuzzy G_δ-set λ is an fuzzy semi-open set in (X,T)

Remark 4.2: In view of the above proposition, one will have the following result: "Fuzzy residual sets in fuzzy globally disconnected and fuzzy semi-P-spaces, are fuzzy semi-open sets."

Remark 4.3: The inter-relations between fuzzy P-spaces, fuzzy semi-P-spaces and fuzzy almost P-spaces can be summarized as follows:

Figure 1. *Inter relations between fuzzy Semi-P-spaces and other fuzzy P-spaces.*

It is to be noted that none of the above relations are reversible. For, consider the following example:

Example 3.2: Let $X = \{a, b, c,d\}$. Let I= [0,1]. The fuzzy sets α, β and γ are defined on X as follows:

α: X→I is defined by $\alpha(a) = 0.6$; $\alpha(b) = 0.5$; $\alpha(c) = 0.4$, $\alpha(d) = 0.7$,

β: X→ I is defined by $\beta(a) = 0.5$; $\beta(b) = 0.7$; $\beta(c) = 0.6$, $\beta(d) = 0.4$.

γ: X→ I is defined by $\gamma(a) = 0.7$; γ (b) = 0.4; γ(c) = 0.7, γ (d) = 0.6.

Then, $T = \{0, \alpha, \beta, \gamma, \alpha\vee\beta, \alpha\vee\gamma, \beta\vee\gamma, \alpha\wedge\beta, \alpha\wedge\gamma, \beta\wedge\gamma, \alpha\vee[\beta\wedge\gamma], \beta\vee[\alpha\wedge\gamma], \gamma\vee[\alpha\wedge\beta], \alpha\wedge[\beta\vee\gamma], \beta\wedge[\alpha\vee\gamma], \gamma\wedge[\alpha\vee\beta], \alpha\vee\beta\vee\gamma, \alpha\wedge\beta\wedge\gamma, 1\}$ is clearly the fuzzy topology on X.

On computation, one can find that $\delta = [\alpha\vee\beta]\wedge[\alpha\vee\gamma]\wedge[\beta\vee\gamma]\wedge(\alpha\vee[\beta\wedge\gamma])\wedge(\beta\vee[\alpha\wedge\gamma])\wedge(\gamma\vee[\alpha\wedge\beta])$ is a fuzzy G_δ-set in (X, T). Also $cl\ int(\delta) = cl(\gamma\vee[\alpha\wedge\beta]) = 1$. Now $\delta \le cl\ int(\delta)$, implies that the fuzzy G_δ-set δ is an fuzzy semi-open set in (X,T). Hence (X, T) is the fuzzy semi-P-space. But (X, T) is not the fuzzy P-space, since the fuzzy G_δ-set δ is not the fuzzy open set in (X, T).

Example 3.3: Let $X = \{a, b, c\}$. Let I = [0,1]. The fuzzy sets α, β, γ, and δ are defined on X as follows:

α: X→I is defined by $\alpha(a) = 0.6$; $\alpha(b) = 0.4$; $\alpha(c) = 0.5$,

β: X→I is defined by $\beta(a) = 0.4$; $\beta(b) = 0.5$; $\beta(c) = 0.6$,

γ: X→ I is defined by $\gamma(a) = 0.5$; $\gamma(b) = 0.7$; $\gamma(c) = 0.4$,

δ: X→ I is defined by $\delta(a) = 0.5$; $\delta(b) = 0.5$; $\delta(c) = 0.5$,

Then, $T = \{0, \alpha, \beta, \gamma, \alpha\vee\beta, \alpha\vee\gamma, \beta\vee\gamma, \alpha\wedge\beta, \alpha\wedge\gamma, \beta\wedge\gamma, \alpha\vee[\beta\wedge\gamma], \beta\vee[\alpha\wedge\gamma], \gamma\vee[\alpha\wedge\beta], \alpha\wedge[\beta\vee\gamma], \beta\wedge[\alpha\vee\gamma], \gamma\wedge[\alpha\vee\beta], \alpha\vee\beta\vee\gamma, \alpha\wedge\beta\wedge\gamma, 1\}$ is the fuzzy topology on X.

On computation one can find that

$$\delta = (\alpha\vee\beta)\wedge(\alpha\vee\gamma)\wedge(\beta\vee\gamma)\wedge(\alpha\vee[\beta\wedge\gamma])$$

$$\wedge(\beta\vee[\alpha\wedge\gamma])\wedge(\gamma\vee[\alpha\wedge\beta]) \text{ and}$$

$$\alpha\wedge\beta\wedge\gamma = (\alpha\wedge\beta)\wedge(\alpha\wedge\gamma)\wedge(\beta\wedge\gamma)\wedge(\alpha\wedge[\beta\vee\gamma])$$

$$\wedge(\beta\wedge[\alpha\vee\gamma])\wedge(\gamma\wedge[\alpha\vee\beta]).$$

Then, δ and $\alpha\wedge\beta\wedge\gamma$ are fuzzy G_δ-sets in (X, T). Also on computation $int(\alpha\wedge\beta\wedge\gamma) = \alpha\wedge\beta\wedge\gamma \neq 0$ and $int(\delta) = (\gamma\vee[\alpha\wedge\beta]) \neq 0$. Hence (X,T) is an fuzzy almost P-space. Now $cl\ int(\delta) = cl(\gamma\wedge[\alpha\vee\beta]) = 1-(\beta\vee[\alpha\wedge\gamma])$ and $\delta \geq cl\ int(\delta)$, implies that the fuzzy G_δ-set δ is not the fuzzy semi-open set in (X,T). Hence (X,T) is not the fuzzy semi-P-space.

Proposition 4.5: If the fuzzy topological space (X,T) is an fuzzy P-space, then (X,T) is an fuzzy semi-P-space.

Proof: Let λ be an fuzzy G_δ-set in (X, T). Since (X,T) is an fuzzy P-space, the fuzzy G_δ-set λ is an fuzzy open set in (X,T). Since each fuzzy open set is an fuzzy semi-open set in fuzzy topological spaces, λ is an fuzzy semi- open set in (X,T) and hence (X,T) is an fuzzy semi-P-space.

Proposition 4.6: If (X,T) is the fuzzy globally disconnected and fuzzy semi-P-space, then (X,T) is the fuzzy P-space.

Proof: Let λ be an fuzzy G_δ-set in (X, T). Since (X,T) is the fuzzy semi-P-space, the fuzzy G_δ-set λ is an fuzzy semi-open set in (X,T). Since (X,T) is the fuzzy globally disconnected space, the fuzzy semi-open set λ is the fuzzy open set in (X,T). Hence the fuzzy G_δ-set λ is an fuzzy open set in (X, T), implies that (X,T) is the fuzzy P-space.

The following proposition gives the condition for the fuzzy hyperconnected and fuzzy open hereditarily irresolvable and fuzzy semi-P-spaces to become fuzzy Baire spaces

Proposition 4.7: If $cl\ [\bigwedge_{i=1}^{\infty}(\lambda_i)] = 1$, where (λ_i)'s are fuzzy G_δ-sets in the fuzzy hyperconnected and fuzzy open hereditarily irresolvable and fuzzy semi-P-space (X,T), then (X, T) is an fuzzy Baire space.

Proof: Suppose that (λ_i)'s are fuzzy G_δ-sets in (X, T). Since (X,T) is the fuzzy semi-P-space, by the proposition 3.1 (ii), (λ_i)'s are fuzzy somewhere dense sets in (X, T). Since (X, T) is the fuzzy hyperconnected space, by the Theorem 2.4, $cl(\lambda_i) = 1$ and then $1 - cl(\lambda_i) = 0$. By the lemma 2.1, $int(1-\lambda_i) = 0$, in (X, T). Since (X,T) is the fuzzy open hereditarily irresolvable space, $int(1-\lambda_i) = 0$, implies that $int\ cl(1-\lambda_i) = 0$ and thus $[1-(\lambda_i)]$'s are fuzzy nowhere dense sets in (X, T). Now the hypothesis $cl\ [\bigwedge_{i=1}^{\infty}(\lambda_i)] = 1$, implies that $1 - cl\ [\bigwedge_{i=1}^{\infty}(\lambda_i)] = 0$, in (X, T) and then by the lemma 2.1, that $int\ [1-\bigwedge_{i=1}^{\infty}(\lambda_i)] = 0$. Thus, $int\ [\bigvee_{i=1}^{\infty}(1-\lambda_i)] = 0$, where $[1-(\lambda_i)]$'s are fuzzy nowhere dense sets in (X, T). Hence (X, T) is the fuzzy Baire space.

Proposition 4.8: If $cl\ [\bigwedge_{i=1}^{\infty}(\lambda_i)] = 1$, where (λ_i)'s are fuzzy G_δ-sets in the fuzzy hyperconnected and fuzzy open hereditarily irresolvable and fuzzy semi-P-space (X,T), then (X, T) is an fuzzy second category space.

Proof: The proof follows from the proposition 4.7 and the theorem 2.6.

Proposition 4.9: If λ is an fuzzy residual set in the fuzzy submaximal and fuzzy semi-P-space (X,T), then λ is an fuzzy semi-open set in (X, T).

Proof: Let λ be an fuzzy residual set in (X, T). Then, $1-\lambda$ is the fuzzy first category set in (X,T) and thus $1-\lambda = \bigvee_{i=1}^{\infty} (\mu_i)$, where $(\mu_i)'s$ are fuzzy nowhere dense sets in (X,T). Since $(\mu_i)'s$ are fuzzy nowhere dense sets in (X,T), $int\ cl(\mu_i) = 0$, in (X,T). Now $int(\mu_i) \le int\ cl\ (\mu_i)$, implies that $int(\mu_i) = 0$. By the lemma 2.1, $cl(1-\mu_i) = 1 - int(\mu_i) = 1 - 0 = 1$ and hence $(1-\mu_i)'s$ are fuzzy dense sets in (X,T). Since (X,T) is the fuzzy submaximal space, $(1-\mu_i)'s$ are fuzzy open sets in (X,T). Thus $(\mu_i)'s$ are fuzzy closed sets in (X,T). Now $1-\lambda = \bigvee_{i=1}^{\infty} (\mu_i)$, where $(\mu_i)'s$ are fuzzy closed sets in (X,T), implies that $1-\lambda$ is the fuzzy F_σ- set in (X,T). Hence λ is the fuzzy G_δ-set in (X,T). Since (X,T) is the fuzzy semi-P-space, the fuzzy G_δ-set λ is an fuzzy semi-open set in (X, T).

Proposition 4.10: If λ is an fuzzy σ-boundary set in the fuzzy submaximal and fuzzy semi-P-space (X,T), then λ is an fuzzy cs dense set in (X, T).

Proof: Let λ be an fuzzy σ-boundary set in (X, T). Since (X,T) is the fuzzy submaximal space, by the theorem 2.7, λ is an fuzzy F_σ -set in (X,T). Since (X,T) is the fuzzy semi-P-space, by the proposition 3.8(i), the fuzzy F_σ-set λ is an fuzzy cs dense set in (X, T).

Proposition 4.11: If λ is an fuzzy G_δ -set in the fuzzy perfectly disconnected and fuzzy semi- P-space (X, T), then $cl(\lambda)$ is an fuzzy open set in (X, T).

Proof: Let λ be an fuzzy G_δ-set in(X, T). Since (X,T) is the fuzzy semi-P-space, the fuzzy G_δ-set λ is an fuzzy semi-open set (X,T) and then $\lambda \le cl\ int(\lambda)$, in (X,T). This implies that $\lambda \le 1-[1-cl\ int(\lambda)]$. Since (X,T) is the fuzzy perfectly disconnected space, for the fuzzy sets λ and $[1-cl\ int(\lambda)]$ with $\lambda \le 1-[1-cl\ int(\lambda)]$, $cl(\lambda) \le 1-cl\ [1-cl\ int(\lambda)]$ and then $cl(\lambda) \le 1-[1-int\ cl\ int(\lambda)]$. Thus, $cl(\lambda) \le int\ cl\ int\ (\lambda) \le int\ cl(\lambda)$. But $int\ cl(\lambda) \le cl(\lambda)$. Thus, $int\ cl(\lambda) = cl(\lambda)$. Hence $cl(\lambda)$ is an fuzzy open set in (X, T).

Proposition 4.12: If λ is an fuzzy σ-nowhere dense set in the fuzzy perfectly disconnected and fuzzy semi- P-space (X,T), then there exists an fuzzy semi-closed set γ in (X,T) such that $\lambda \le int\ (\gamma)$.

Proof: Let λ be an fuzzy σ-nowhere dense set in (X, T). Since (X,T) is the fuzzy perfectly disconnected by the theorem 2.8, there exists an fuzzy G_δ-set η in (X,T) such that $\lambda \le int\ (1-\eta)$. Since (X,T) is the fuzzy semi-P-space, the fuzzy G_δ-set η is an fuzzy semi-open set (X,T) and hence $1-\eta$ is the fuzzy semi-closed in (X,T). Let $\gamma = 1-\eta$. Thus, for the fuzzy σ-nowhere dense set λ in (X, T), there exists an fuzzy semi-closed set γ in (X,T) such that $\lambda \le int\ (\gamma)$.

The following proposition gives the condition for the fuzzy semi-P-spaces to become fuzzy strongly irresolvable spaces.

Proposition 4.13: If each fuzzy dense set is an fuzzy G_δ-set in the fuzzy semi- P-space (X,T), then (X,T) is the fuzzy strongly irresolvable space.

Proof: Let λ be an fuzzy dense set in (X,T). That is, $cl\ (\lambda) = 1$, in (X,T). By the hypothesis, λ is an fuzzy G_δ-set in (X,T). Since (X,T) is the fuzzy semi-P-space, the fuzzy G_δ-set λ is an fuzzy semi-open set in (X,T) and thus $\lambda \le cl\ int\ (\lambda)$, in (X,T). This implies that $cl(\lambda) \le cl\ (cl\ int(\lambda))$. Then, $1 \le cl\ int\ (\lambda)$. That is, $cl\ int\ (\lambda) = 1$, in (X,T). Hence, for the fuzzy dense set λ in (X,T), $cl\ int\ (\lambda) = 1$, in (X, T), implies that (X, T) is the fuzzy strongly irresolvable space.

Proposition 4.14: If λ is an fuzzy co -σ -boundary set in the fuzzy perfectly disconnected and fuzzy semi- P-space (X,T), then there exists an fuzzy semi-open set δ in (X,T) such that $\lambda \le \delta$.

Proof: Let λ be an fuzzy co-σ-boundary set in (X,T). Since (X,T) is the fuzzy perfectly disconnected by the theorem 2.9, there exists an fuzzy G_δ-set δ in (X,T) such that $\lambda \le \delta$. Since (X,T) is the fuzzy semi-P-space, the fuzzy G_δ -set δ is an fuzzy semi-open set (X,T). Hence for the fuzzy co - σ - boundary set λ in (X,T), there exists an fuzzy semi-open set δ in (X,T) such that $\lambda \le \delta$.

Proposition 4.15: If λ is an fuzzy G_δ -set in the fuzzy perfectly disconnected and fuzzy semi- P-space (X, T), then λ is an fuzzy pre-open set in (X, T).

Proof: Let λ be an fuzzy G_δ-set in (X,T). Since (X,T) is the fuzzy semi-P-space, the fuzzy G_δ-set λ is an fuzzy semi-open set in (X,T) and then $1-\lambda$ is the fuzzy semi-closed set in (X,T). Since (X,T) is the fuzzy perfectly disconnected space, by the theorem 2.10, $1-\lambda$ is the fuzzy pre-closed set in (X, T) and hence λ is an fuzzy pre-open set in (X, T).

Proposition 4.16: If λ is an fuzzy F_σ -set in the fuzzy hyperconnected and fuzzy semi- P-space (X, T), then $int\ (\lambda) = 0$, in (X,T).

Proof: Let λ be an fuzzy F_σ-set in (X,T). Since (X,T) is the fuzzy semi-P-space, by the proposition 3.8 (i), λ is an fuzzy cs dense set in(X, T). Then, $1-\lambda$ is an fuzzy somewhere dense set in(X, T). Since (X, T)is the fuzzy hyperconnected space, by the theorem 2.4, $1-\lambda$ is the fuzzy dense set in (X, T) and then $cl(1-\lambda) = 1$, This implies, by the lemma 2.1, that $1 - int(\lambda) = 1$ and then $int(\lambda) = 0$, in (X, T).

Proposition 4.17: If λ is an fuzzy F_σ -set in the fuzzy hyperconnected and fuzzy semi-p-space (X, T), then λ is an fuzzy σ-nowhere dense set in (X,T).

Proof: Let λ be an fuzzy F_σ–set in (X,T). Since (X,T) is the fuzzy hyper-connected and fuzzy semi-P-space, by the proposition 4.16, $int\ (\lambda) = 0$, in (X, T). Hence λ is an fuzzy F_σ-set in (X,T) such that $int(\lambda) = 0$ and hence λ is an fuzzy σ-nowhere dense set in (X,T).

Remark 4.4: In view of the above proposition, one will have the following result: "Fuzzy F_σ -sets in fuzzy hyperconnected and fuzzy semi- P-space s are fuzzy σ - nowhere dense sets".

The following proposition shows that fuzzy hyperconnected and fuzzy semi-P-spaces, are fuzzy irresolvable spaces.

Proposition 4.18: If (X,T) is the fuzzy hyperconnected and fuzzy semi-P-space, then (X,T) is the fuzzy irresolvable space.

Proof: Let λ be an fuzzy F_σ–set in (X,T). Since (X,T) is the fuzzy hyperconnected and fuzzy semi-P-space, by the proposition 4.16, $int\ (\lambda) = 0$, in (X, T). Then, $cl\ (1-\lambda) = 1 - int\ (\lambda) = 1$. Since (X,T) is the fuzzy semi-P-space, by the proposition 3.8 (ii), $cl(\lambda) \neq 1$, in (X, T). Thus, for the fuzzy dense set $1-\lambda$ in (X, T), $cl\ [1-(1-\lambda)] = cl(\lambda) \neq 1$, in (X, T). Hence (X,T) is the fuzzy irresolvable space.

Proposition 4.19: If λ is an fuzzy σ-boundary set in the fuzzy hyperconnected and fuzzy semi- P-space (X, T), then λ is an fuzzy σ-nowhere dense set in (X,T).

Proof: Let λ be an fuzzy σ-boundary set in (X,T). Then, by the theorem 2.7, λ is an fuzzy F_σ-set in (X,T). Since (X,T) is the fuzzy hyperconnected and fuzzy semi-P-spaces, by the proposition 4.16, $int\ (\lambda) = 0$, in (X, T). Hence λ is an fuzzy F_σ -set in (X,T) such that $int(\lambda) = 0$ and hence λ is the fuzzy σ-nowhere dense set in (X,T).

Remark 4.5: In view of the above proposition, one will have the following result: "Fuzzyσ-boundary sets in fuzzy hyperconnected and fuzzy semi-P-spaces are fuzzy σ-nowhere dense sets".

The following proposition gives the condition for fuzzy G_δ-sets in fuzzy globally disconnected and fuzzy semi-P-spaces to become fuzzy simply open sets.

Proposition 4.20: If λ is the fuzzy dense and fuzzy G_δ -set in the fuzzy globally disconnected and fuzzy semi-P-space (X,T), then λ is the fuzzy simply open set in (X,T).

Proof: Let λ be an fuzzy dense and fuzzy G_δ-set in (X,T). Since (X,T) is the fuzzy semi-P-space, the fuzzy G_δ-set λ is the fuzzy semi-open set in (X,T). Thus, λ is the fuzzy semi-open and fuzzy dense set in (X,T). Since (X,T) is the fuzzy globally disconnected space, by the theorem 2.12, λ is the fuzzy simply open set in (X,T).

Proposition4.21: If λ is the fuzzy residual set in the fuzzy globally disconnected and fuzzy semi-P-space (X,T), then $cl\ int(\lambda) = cl\ (\lambda)$, in (X,T).

Proof: Let λ be an fuzzy residual set in (X,T). Since (X,T) is an fuzzy globally disconnected space, by the theorem 2.5, λ is the fuzzy G_δ-set in (X,T). Since (X,T) is the fuzzy semi-P-space, for the fuzzy G_δ-set λ in (X,T), by the proposition 3.1(iii), $cl\ int(\lambda) = cl\ (\lambda)$, in (X,T).

Proposition 4.22: If λ is the fuzzy F_σ-set in the fuzzy globally disconnected and fuzzy semi-P-space (X,T), then,

(i). $cl(1-\lambda) = 1 - cl\ int(\lambda)$, in (X,T).

(ii). $cl\ int(\lambda) = int\ (\lambda)$, in (X,T).

Proof:

(i). Let λ be the fuzzy F_σ-set in (X,T). Then, $1-\lambda$ is the fuzzy G_δ-set in (X,T). Since (X,T) is the fuzzy semi-P-space, the fuzzy G_δ-set $(1-\lambda)$ is the fuzzy semi-open set in (X,T) and thus λ is the fuzzy semi-closed set in (X,T). Since (X,T) is the fuzzy globally disconnected space, by the theorem 2.1, for the fuzzy semi-closed set λ in (X, T), $cl(1-\lambda) = 1 - cl\ int\ (\lambda)$, in (X,T).

(ii). From (i), for the fuzzy F_σ-set λ in (X, T), $cl\ (1-\lambda) = 1 - cl\ int\ (\lambda)$, in (X,T). Then, $1 - int\ (\lambda) = 1 - cl\ int\ (\lambda)$ and thus $int\ (\lambda) = cl\ int\ (\lambda)$, in (X,T).

The following proposition gives the condition for the fuzzy globally disconnected and fuzzy semi-P-spaces to become fuzzy extremally disconnected spaces.

Proposition 4.23: If each fuzzy open set is the fuzzy F_σ – set in the fuzzy globally disconnected and fuzzy semi-P-space (X,T), then (X,T) is an fuzzy extremally disconnected space.

Proof: Let λ be an fuzzy open set in (X,T). By the hypothesis, λ is an fuzzy F_σ -set in the fuzzy globally disconnected and fuzzy semi-P-space (X,T). Then, by the proposition 4.22 (ii), $cl\ int(\lambda) = int\ (\lambda)$, in (X,T). Since λ is the fuzzy open set in (X,T), $int\ (\lambda) = \lambda$ and then $cl\ (\lambda) = \lambda \in T$. Hence, for the fuzzy open set λ in (X,T), $cl\ (\lambda) \in T$ implies that (X,T) is the fuzzy extremally disconnected space.

The following proposition gives the condition for the fuzzy *GID* spaces to become fuzzy semi-P-spaces.

Proposition 4.24: If each fuzzy G_δ-set is an fuzzy dense set in the fuzzy *GID* space (X,T), then (X,T) is the fuzzy semi-P-space.

Proof: Let λ be an fuzzy G_δ -set in (X,T). By the hypothesis, λ is an fuzzy dense set in (X,T). Then, λ is an fuzzy dense and fuzzy G_δ-set in (X,T). Since (X,T) is the fuzzy *GID* space, by the theorem 2.11, the fuzzy G_δ-set λ is the fuzzy semi-open set in (X,T). This implies that (X, T) is an fuzzy semi-P-space.

Proposition 4.25: If λ is an fuzzy first category set in the fuzzy submaximal and fuzzy semi-P-space (X,T), then λ is the fuzzy semi-closed set in (X,T).

Proof: Let λ be an fuzzy first category set in (X,T). Then, $1-\lambda$ is an fuzzy residual set in (X,T). Since (X,T) is the fuzzy submaximal and fuzzy semi-P-space, by the proposition 4.9, $1-\lambda$ is the fuzzy semi-open set in (X,T). Hence λ is the fuzzy semi-closed set in (X,T).

Remark 4.6: In view of the above proposition, one will have the following result: "Fuzzy first category sets in fuzzy submaximal and fuzzy semi- P-spaces are fuzzy semi-closed sets".

5. Conclusion

In this paper, the notion of fuzzy semi-P-space is introduced by means of fuzzy semi-openness of fuzzy G_δ-sets. It is established that the fuzzy G_δ -sets are fuzzy somewhere dense sets, fuzzy β-open sets and fuzzy σ-nowhere dense sets are fuzzy semi-closed sets in fuzzy semi-P-spaces. It is established that the class of fuzzy semi-P-spaces lies between the classes of fuzzy P-spaces and fuzzy almost P-spaces. Also it is obtained that fuzzy F_σ -sets in fuzzy semi-P-spaces are fuzzy cs dense sets but not fuzzy dense sets and fuzzy F_σ-sets in fuzzy hyperconnected and fuzzy semi-P-spaces, are fuzzy σ-nowhere dense sets. Also it is obtained that fuzzy residual sets in fuzzy globally disconnected and fuzzy semi-P-spaces, are fuzzy semi-open sets. The conditions for the fuzzy semi-P-spaces to become fuzzy P-spaces and for the fuzzy hyperconnected and fuzzy semi-P-spaces to become fuzzy σ-Baire spaces, are obtained. Also the conditions for the fuzzy semi-P-spaces to become fuzzy strongly irresolvable spaces and for the fuzzy hyperconnected, fuzzy open hereditarily irresolvable and fuzzy semi-P-spaces to become fuzzy Baire spaces are obtained. The conditions under which the fuzzy *GID* spaces become fuzzy semi-P-spaces and fuzzy globally disconnected and fuzzy semi-P-spaces become fuzzy extremally disconnected spaces, are also obtained in this paper.

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